

## STEP III, 2006, Q8

- 8  $\Delta$  is an operation that takes polynomials in  $x$  to polynomials in  $x$ ; that is, given any polynomial  $h(x)$ , there is a polynomial called  $\Delta h(x)$  which is obtained from  $h(x)$  using the rules that define  $\Delta$ . These rules are as follows:

(i)  $\Delta x = 1$ ;

(ii)  $\Delta(f(x) + g(x)) = \Delta f(x) + \Delta g(x)$  for any polynomials  $f(x)$  and  $g(x)$ ;

(iii)  $\Delta(\lambda f(x)) = \lambda \Delta f(x)$  for any constant  $\lambda$  and any polynomial  $f(x)$ ;

(iv)  $\Delta(f(x)g(x)) = f(x)\Delta g(x) + g(x)\Delta f(x)$  for any polynomials  $f(x)$  and  $g(x)$ .

Using these rules show that, if  $f(x)$  is a polynomial of degree zero (that is, a constant), then  $\Delta f(x) = 0$ . Calculate  $\Delta x^2$  and  $\Delta x^3$ .

Prove that  $\Delta h(x) \equiv \frac{dh(x)}{dx}$  for any polynomial  $h(x)$ . You should make it clear whenever you use one of the above rules in your proof.



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