STEP III, 2006, Q7 MS

7 (i)	Express sinhx in terms of exponentials, factorise and solve to get $u = -e^x$ or $u = e^{-x}$ (or $-\cosh x \pm \sinh x$). Use both of these as equal to $\frac{dy}{dx}$ and integration to get alternative solutions $y = -e^{\pm x} + c.$ From the given conditions the particular integral is $y = 1 - e^{-x}$.
(ii)	Solve the quadratic as before to get either $u = \frac{-1 \pm \cosh y}{\sinh y}$ (or equivalent) $\Rightarrow \frac{dx}{dy} = \frac{\sinh y}{-1 \pm \cosh y}$ $\Rightarrow x = \ln(\cosh y - 1) + c_1$ or $x = -\ln(\cosh y + 1) + c_2$ Only the first can satisfy the conditions $x = 0,\ y = 0$; then we have $x = \ln \frac{2}{1 + \cosh y}$ $\Rightarrow \cosh y = 2e^{-x} - 1$ This is undefined for $x > 0$. For $x \to -\infty \Rightarrow \cosh y \to \infty$, and there will be two branches, corresponding to $y \to \pm\infty$, as cosh is an even function. So $x \to -\infty \Rightarrow \cosh y \to \infty \Rightarrow y \to \infty \Rightarrow e^y$ □ $4e^{-x} \Rightarrow y = -x + \ln 4$ in one case, and similarly $y = x - \ln 4$ in the other.