

STEP III, 2006, Q7 MS

7 (i)	<p>Express $\sinh x$ in terms of exponentials, factorise and solve to get $u = -e^x$ or $u = e^{-x}$ (or $-\cosh x \pm \sinh x$).</p> <p>Use both of these as equal to $\frac{dy}{dx}$ and integration to get alternative solutions</p> $y = -e^{\pm x} + c.$ <p>From the given conditions the particular integral is</p> $y = 1 - e^{-x}.$
(ii)	<p>Solve the quadratic as before to get either</p> $u = \frac{-1 \pm \cosh y}{\sinh y} \text{ (or equivalent)}$ $\Rightarrow \frac{dx}{dy} = \frac{\sinh y}{-1 \pm \cosh y}$ $\Rightarrow x = \ln(\cosh y - 1) + c_1$ <p>or $x = -\ln(\cosh y + 1) + c_2$</p> <p>Only the first can satisfy the conditions $x = 0, y = 0$; then we have</p> $x = \ln \frac{2}{1 + \cosh y}$ $\Rightarrow \cosh y = 2e^{-x} - 1$ <p>This is undefined for $x > 0$.</p> <p>For $x \rightarrow -\infty \Rightarrow \cosh y \rightarrow \infty$, and there will be two branches, corresponding to $y \rightarrow \pm\infty$, as \cosh is an even function.</p> <p>So $x \rightarrow -\infty \Rightarrow \cosh y \rightarrow \infty \Rightarrow y \rightarrow \infty \Rightarrow e^y \approx 4e^{-x} \Rightarrow y = -x + \ln 4$ in one case, and similarly $y = x - \ln 4$ in the other.</p>



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