

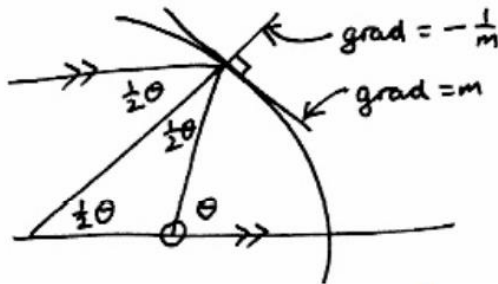
### STEP III, 2006, Q6 MS

6

$$x = r \cos \theta, y = r \sin \theta, r = r(\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

and result follows.



Gradient of the normal is  $\tan \frac{\theta}{2} = t$ , say. Then we have

$$t = -\frac{\frac{dr}{d\theta} - r \tan \theta}{\frac{dr}{d\theta} \tan \theta + r}, \tan \theta = \frac{2t}{1-t^2}$$

This reduces to

$$\frac{dr}{d\theta} = rt$$

$$\Rightarrow \ln r = \int \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} d\theta$$

$$= -2 \ln \left[ c \cos \frac{\theta}{2} \right]$$

$$\Rightarrow \frac{2}{c^2 r} = 1 + \cos \theta \quad (\text{using } 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2})$$

This corresponds to the standard equation of a parabola in polars.



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)