

## STEP III, 2006, Q5

- 5 Show that the distinct complex numbers  $\alpha$ ,  $\beta$  and  $\gamma$  represent the vertices of an equilateral triangle (in clockwise or anti-clockwise order) if and only if

$$\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta = 0.$$

Show that the roots of the equation

$$z^3 + az^2 + bz + c = 0 \quad (*)$$

represent the vertices of an equilateral triangle if and only if  $a^2 = 3b$ .

Under the transformation  $z = pw + q$ , where  $p$  and  $q$  are given complex numbers with  $p \neq 0$ , the equation (\*) becomes

$$w^3 + Aw^2 + Bw + C = 0. \quad (**)$$

Show that if the roots of equation (\*) represent the vertices of an equilateral triangle, then the roots of equation (\*\*) also represent the vertices of an equilateral triangle.



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