

### STEP III, 2006, Q4 MS

4	<p>Let <math>x = y</math> and deduce first result.</p> $2f(x) = f(2x)$ $\Rightarrow 2f'(x) = 2f'(2x)$ $\Rightarrow 2f''(x) = 4f''(2x)$ <p>then put <math>x = 0</math> to get <math>f(0) = 0, f''(0) = 0</math>.</p> <p>Similarly all higher order derivatives are zero, so by Maclaurin the most general function is <math>cx</math>, where <math>c</math> is a constant.</p>
(i)	<p>Use properties of logs to show that <math>G(x) + G(y) = G(x + y)</math>.</p> <p>Deduce that <math>g(x) = e^{cx}</math>.</p>
(ii)	<p>Show that <math>H(u) + H(v) = H(u + v)</math></p> <p>so <math>h(x) = c \ln x</math>.</p>
(iii)	<p>Let <math>T(x) = t(\tan x)</math>.</p> <p>Deduce that <math>t(x) = c \arctan x</math>.</p>



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