

STEP III, 2006, Q4

- 4 The function f satisfies the identity

$$f(x) + f(y) \equiv f(x + y) \quad (*)$$

for all x and y . Show that $2f(x) \equiv f(2x)$ and deduce that $f''(0) = 0$. By considering the Maclaurin series for $f(x)$, find the most general function that satisfies (*).

[Do not consider issues of existence or convergence of Maclaurin series in this question.]

- (i) By considering the function G , defined by $\ln(g(x)) = G(x)$, find the most general function that, for all x and y , satisfies the identity

$$g(x)g(y) \equiv g(x + y).$$

- (ii) By considering the function H , defined by $h(e^u) = H(u)$, find the most general function that satisfies, for all positive x and y , the identity

$$h(x) + h(y) \equiv h(xy).$$

- (iii) Find the most general function t that, for all x and y , satisfies the identity

$$t(x) + t(y) \equiv t(z),$$

$$\text{where } z = \frac{x + y}{1 - xy}.$$



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