STEP III, 2006, Q2 MS

2 (i)	First "show" by change of variable $\theta = -\phi$ (say). Then

	$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2\theta}{1+\sin\theta\sin 2\alpha} d\theta + \int_{-\pi/2}^{\pi/2} \frac{\cos^2\theta}{1-\sin\theta\sin 2\alpha} d\theta$ $= \int_{-\pi/2}^{\pi/2} \frac{2}{\sec^2\theta - \tan^2\theta\sin^2 2\alpha} d\theta$ and next "show" follows.
(ii)	$J = \sec 2\alpha \int_{-\pi/2}^{\pi/2} \frac{1}{1+(\cos 2\alpha \tan\theta)^2} \cos 2\alpha \sec^2\theta d\theta$ $= \sec 2\alpha \int_{-\pi/2}^{\pi/2} \frac{1}{1+u^2} du$ (since $\cos 2\alpha > 0$) $= \pi \sec 2\alpha$
(iii)	$I\sin^2 2\alpha + J\cos^2 2\alpha = \pi$. Result follows after use of (ii).
(iv)	In this case, $\cos 2\alpha < 0$, so $J = -\pi \sec 2\alpha$. Then $I = \frac{1}{2}\pi \operatorname{cosec}^2\alpha$