

STEP III, 2006, Q2 MS

2 (i)	First “show” by change of variable $\theta = -\phi$ (say). Then
	$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \theta}{1 + \sin \theta \sin 2\alpha} d\theta + \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \theta}{1 - \sin \theta \sin 2\alpha} d\theta$ $= \int_{-\pi/2}^{\pi/2} \frac{2}{\sec^2 \theta - \tan^2 \theta \sin^2 2\alpha} d\theta$ <p>and next “show” follows.</p>
(ii)	$J = \sec 2\alpha \int_{-\pi/2}^{\pi/2} \frac{1}{1 + (\cos 2\alpha \tan \theta)^2} \cos 2\alpha \sec^2 \theta d\theta$ $= \sec 2\alpha \int_{-\pi/2}^{\pi/2} \frac{1}{1 + u^2} du \quad (\text{since } \cos 2\alpha > 0)$ $= \pi \sec 2\alpha$
(iii)	$I \sin^2 2\alpha + J \cos^2 2\alpha = \pi .$ <p>Result follows after use of (ii).</p>
(iv)	<p>In this case, $\cos 2\alpha < 0$, so $J = -\pi \sec 2\alpha$.</p> <p>Then $I = \frac{1}{2} \pi \operatorname{cosec}^2 \alpha$</p>



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