

## STEP III, 2006, Q2

2 Let

$$I = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{\cos^2 \theta}{1 - \sin \theta \sin 2\alpha} d\theta \quad \text{and} \quad J = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{\sec^2 \theta}{1 + \tan^2 \theta \cos^2 2\alpha} d\theta$$

where  $0 < \alpha < \frac{1}{4}\pi$ .

- (i) Show that  $I = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{\cos^2 \theta}{1 + \sin \theta \sin 2\alpha} d\theta$  and hence that  $2I = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{2}{1 + \tan^2 \theta \cos^2 2\alpha} d\theta$ .
- (ii) Find  $J$ .
- (iii) By considering  $I \sin^2 2\alpha + J \cos^2 2\alpha$ , or otherwise, show that  $I = \frac{1}{2}\pi \sec^2 \alpha$ .
- (iv) Evaluate  $I$  in the case  $\frac{1}{4}\pi < \alpha < \frac{1}{2}\pi$ .



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