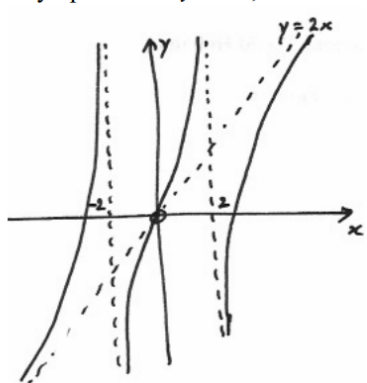


STEP III, 2006, Q1 MS

1	$y = \frac{2x(x^2 - 5)}{x^2 - 4}$ $= 2x - \frac{2x}{(x-2)(x+2)}$ <p>Asymptotes are $y = 2x$, $x = \pm 2$.</p>  $\frac{dy}{dx} = 2 - \frac{2(x-2)(x+2) - 4x^2}{(x-2)^2(x+2)^2}$ <p>(or equivalent). Equation of the tangent at O is $y = \frac{5x}{2}$.</p>
(i)	$3x(x^2 - 5) = (x^2 - 4)(x + 3)$ $\Leftrightarrow \frac{2x(x^2 - 5)}{x^2 - 4} = \frac{2x}{3} + 2 \quad (x \neq \pm 2)$ <p>$y = \frac{2}{3}x + 2$ cuts the sketched curve in three points, so three roots.</p>
(ii)	$4x(x^2 - 5) = (x^2 - 4)(5x - 2)$ $\Leftrightarrow \frac{2x(x^2 - 5)}{x^2 - 4} = \frac{5x}{2} - 1 \quad (x \neq \pm 2)$ <p>$y = \frac{5x}{2} - 1$ passes through the intersection of $x = 2$ and $y = 2x$ and is parallel to $y = \frac{5x}{2}$ so just one root.</p>
(iii)	$4x^2(x^2 - 5)^2 = (x^2 - 4)^2(x^2 + 1)$ $\Leftrightarrow \frac{2x(x^2 - 5)}{x^2 - 4} = \pm \sqrt{x^2 + 1} \quad (x \neq \pm 2)$ <p>$y = \pm \sqrt{x^2 + 1}$ has two branches with asymptotes $y = \pm x$, so there are six roots.</p>



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