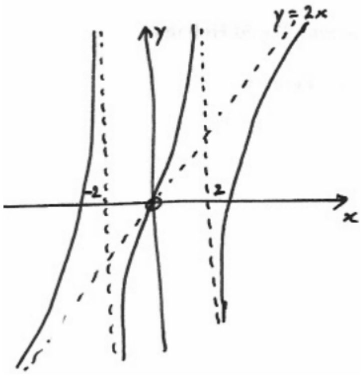
STEP III, 2006, Q1 MS

1	$y=\frac{2x(x^2-5)}{x^2-4}$ $=2x-\frac{2x}{(x-2)(x+2)}$ Asymptotes are $y=2x$, $x=\pm 2$.  $\frac{dy}{dx}=2-\frac{2(x-2)(x+2)-4x^2}{(x-2)^2(x+2)^2}$ (or equivalent). Equation of the tangent at O is $y=\frac{5x}{2}$.
(i)	$3x(x^2-5)=(x^2-4)(x+3)$ $\Leftrightarrow \frac{2x(x^2-5)}{x^2-4}=\frac{2x}{3}+2 \quad (x\neq\pm 2)$ $y=\frac{2}{3}x+2$ cuts the sketched curve in three points, so three roots.
(ii)	$4x(x^2-5)=(x^2-4)(5x-2)$ $\Leftrightarrow \frac{2x(x^2-5)}{x^2-4}=\frac{5x}{2}-1 \quad (x\neq\pm 2)$ $y=\frac{5x}{2}-1$ passes through the intersection of $x=2$ and $y=2x$ and is parallel to $y=\frac{5x}{2}$ so just one root.
(iii)	$4x^2(x^2-5)^2=(x^2-4)^2(x^2+1)$ $\Leftrightarrow \frac{2x(x^2-5)}{x^2-4}=\pm\sqrt{(x^2+1)} \quad (x\neq\pm 2)$ $y=\pm\sqrt{(x^2+1)}$ has two branches with asymptotes $y=\pm x$, so there are six roots.