STEP III, 2006, Q14 MS

14	$E[aX_1 + bX_2] = aE[X_1] + bE[X_2]$ $E[X_1X_2] = E[X_1]E[X_2]$ $E[P] = 2\mu_1 + 2\mu_2$ $E[P^2] = 4E[X_1^2] + 8E[X_1]E[X_2] + 4E[X_2^2]$ $E[X_1^2] = \mu_1^2 + \sigma_1^2$ $\text{var}[P] = 4(\sigma_1^2 + \sigma_2^2)$ The standard deviation is the square root of that expression. $E[A] = \mu_1\mu_2$ $E[A^2] = \mu_1^2\mu_2^2$ $\text{var}[A] = \sigma_1^2\mu_2^2 + \sigma_2^2\mu_1^2 + \sigma_1^2\sigma_2^2$ Again the standard deviation is the square root. Now find $\text{cov}[P, A] = 2\mu_2\sigma_1^2 + 2\mu_1\sigma_2^2$ This is not zero (as independence would imply) with given conditions. Similarly $\text{cov}[Z, A] = 2\sigma_1^2\mu_2 + 2\sigma_2^2\mu_1 - \alpha(\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2)$ That too is non-zero when α is not the excluded value. We consider the exceptional case with the given information. We have $\mu_1 = \mu_2 = 2, \sigma_1^2 = \sigma_2^2 = 1, \alpha = \frac{8}{9}$. Only three values of A are possible - 1, 3 and 9 - and they correspond to unique values of Z. Dependence can be shown by considering, for example, $pr\left(Z = \frac{28}{9}\right) = \frac{1}{4},\ pr\left(Z = \frac{28}{9} \middle| A = 3\right) = 0$.