

STEP III, 2006, Q14

- 14 For any random variables X_1 and X_2 , state the relationship between $E(aX_1 + bX_2)$ and $E(X_1)$ and $E(X_2)$, where a and b are constants. If X_1 and X_2 are independent, state the relationship between $E(X_1X_2)$ and $E(X_1)$ and $E(X_2)$.

An industrial process produces rectangular plates. The length and the breadth of the plates are modelled by independent random variables X_1 and X_2 with non-zero means μ_1 and μ_2 and non-zero standard deviations σ_1 and σ_2 , respectively. Using the results in the paragraph above, and without quoting a formula for $\text{Var}(aX_1 + bX_2)$, find the means and standard deviations of the perimeter P and area A of the plates. Show that P and A are not independent.

The random variable Z is defined by $Z = P - \alpha A$, where α is a constant. Show that Z and A are not independent if

$$\alpha \neq \frac{2(\mu_1\sigma_2^2 + \mu_2\sigma_1^2)}{\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2}.$$

Given that X_1 and X_2 can each take values 1 and 3 only, and that they each take these values with probability $\frac{1}{2}$, show that Z and A are not independent for any value of α .



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