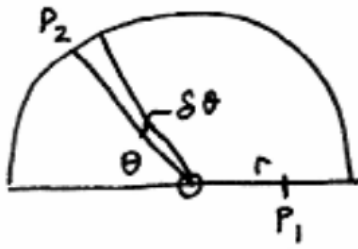


### STEP III, 2006, Q13 MS

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There are three cases to consider: (i) both on the circumference, (ii)  $P_1$  on the diameter and  $P_2$  on the circumference, and (iii) vice versa.

For case (i), if  $P_1$  lies in the arc  $(\alpha, \alpha + \delta\alpha)$   $P_2$  lies in the arc  $(\theta, \theta + \delta\theta)$ , with probability  $\frac{\delta\theta}{\pi+2}$ , the area is  $\frac{1}{2}|r|\sin\theta$ . The expected area given  $P_1$

lies in the arc  $(\alpha, \alpha + \delta\alpha)$  is by integration  $\frac{1}{\pi+2}$ .

For case (ii), if  $P_1$  lies in  $(r, r + \delta r)$  and  $P_2$  lies in the arc  $(\theta, \theta + \delta\theta)$ , with probability  $\frac{\delta\theta}{\pi+2}$ , the area is  $\frac{1}{2}|r|\sin\theta$ . The expected area given  $P_1$  lies in

$(r, r + \delta r)$  from  $O$  is by integration  $\frac{|r|}{\pi+2}$ .

Case (iii) is essentially the same as case (ii).

Thus the expected area is

$$\int_0^\pi \frac{1}{\pi+2} \cdot \frac{1}{\pi+2} d\alpha + 2 \int_{-1}^1 \frac{|r|}{\pi+2} \cdot \frac{1}{\pi+2} dr$$

where the first integral corresponds to case (i) and the second to (ii) and (iii).

This evaluates to the answer given.



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