

STEP III, 2006, Q12 MS

- 12 Model each tourist as trial with success probability $\frac{1}{2}$. If X is the number of potential passengers $X \square Bin(1024, \frac{1}{2})$, ie $N(512, 16^2)$ approximately. Lost profit corresponds to $X > 480$. Hence if L is the loss, we have

$$\begin{aligned}
 E[L] &= \sum_{k=1}^{32} kpr(X = 480 + k) + 32pr(X > 512) \\
 &= \sum_{k=1}^{32} k \cdot \frac{1}{16} \cdot \phi\left(-2 + \frac{k}{16}\right) + 16 \\
 &\approx \int_0^{32} \frac{x}{16} \phi\left(-2 + \frac{x}{16}\right) dx + 16 \\
 &= \int_0^{32} \frac{x}{16} \cdot \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-32)^2}{512}\right) dx + 16
 \end{aligned}$$

Now use substitution to show that this evaluates to

$$\frac{16}{\sqrt{2\pi}}(e^{-2} - 1) + 32\Phi(2).$$

In the course of year the expectation is 50 times that figure, so that is the maximum tolerable licence fee.



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