STEP III, 2006, Q11 MS

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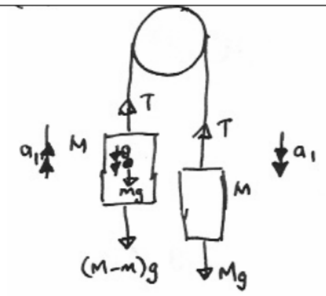


The equations of motion are

$T-(M-m)g=(M-m)a_1$

$Mg-T=Ma_1$

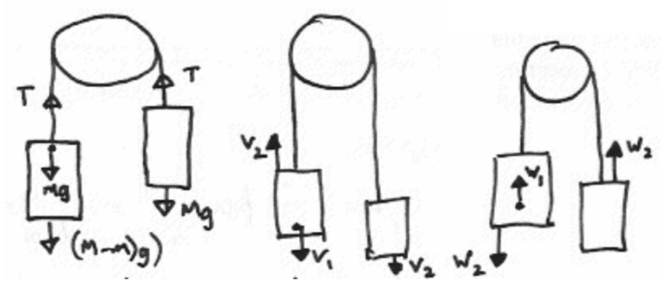
$\Rightarrow a_1=\dfrac{mg}{2M-m}$

Now consider relative motion of the tile with acceleration $(g+a_1)$.

If the time of the first stage is t_1, $s=ut+\frac{1}{2}at^2$ gives

$t_1=\sqrt{\dfrac{(2M-m)h}{Mg}}$

and then for the absolute motion of the tile $v=u+at$ gives the required final velocity.



The middle diagram shows the situation before the impact and the third after. The forces acting on the left-hand system (lift plus tile) are exactly the same as those on the right, so the changes in momentum must be equal in the first stage of the motion. Thus given that all is stationary initially

$-(M-m)v_2+mv_1=Mv_2$

$\Rightarrow v_2=\dfrac{m}{2M-m}v_1=\alpha v_1$ (*), say.

In the collision, the equality of impulsive tensions given means that the change in momentum on one side equals change in momentum on the other. Hence we have

$-Mw_2-Mv_2=-mw_1-mv_1+(M-m)(w_2+v_2)$

$\Rightarrow w_2+v_2=\alpha(w_1+v_1)$

Thus from the two last equations

$w_2=\alpha w_1$ (**).

Newton's experimental law and the two asterisked equations give

$w_1=ev_1$.

	Then the change of energy in the collision $\frac{1}{2}(2M-m)(v_2^2-w_2^2)+\frac{1}{2}m(v_1^2-w_1^2)$ simplifies to the required expression when the above relations are substituted. Loss of energy of a tile dropping to the floor of a fixed lift and bouncing would be just the same.