

STEP III, 2006, Q10 MS

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Use conservation of angular momentum for the first result.

Use conservation of energy to derive

$$v^2 = \frac{k^2 + a^2}{k^2} \Omega^2 - (k^2 + r^2) \omega^2$$

and so by use of the first result and $v = -\frac{dr}{dt}$

second result follows.

Now use $\omega = \frac{d\theta}{dt}$ and $\frac{dr}{d\theta} = \frac{dr}{dt} \bigg/ \frac{d\theta}{dt}$ and the two displayed result to derive

the third.

The suggested substitution transforms the third displayed equation to

$$\frac{du}{d\theta} = \sqrt{1+u^2}.$$

Invert and integrate to get the desired result.

Hence $r = \frac{k}{\sinh(\theta + \alpha)}.$

As $\theta \rightarrow \infty, r \rightarrow 0+,$ but $r = 0$ is impossible.



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