STEP III, 2006, Q10 MS

10	Use conservation of angular momentum for the first result. Use conservation of energy to derive $v^2 = \frac{k^2 + a^2}{k^2}\Omega^2 - (k^2 + r^2)\omega^2$ and so by use of the first result and $v = -\frac{dr}{dt}$ second result follows. Now use $\omega = \frac{d\theta}{dt}$ and $\frac{dr}{d\theta} = \frac{dr}{dt} \Big/ \frac{d\theta}{dt}$ and the two displayed result to derive the third. The suggested substitution transforms the third displayed equation to $\frac{du}{d\theta} = \sqrt{1+u^2}$. Invert and integrate to get the desired result. Hence $r = \frac{k}{\sinh(\theta + \alpha)}$. As $\theta \to \infty, r \to 0+$, but $r = 0$ is impossible.