

STEP III, 2005, Q8

- 8 In this question, a and c are distinct non-zero complex numbers. The complex conjugate of any complex number z is denoted by z^* .

Show that

$$|a - c|^2 = aa^* + cc^* - ac^* - ca^*$$

and hence prove that the triangle OAC in the Argand diagram, whose vertices are represented by 0 , a and c respectively, is right angled at A if and only if $2aa^* = ac^* + ca^*$.

Points P and P' in the Argand diagram are represented by the complex numbers ab and $\frac{a}{b^*}$, where b is a non-zero complex number. A circle in the Argand diagram has centre C and passes through the point A , and is such that OA is a tangent to the circle. Show that the point P lies on the circle if and only if the point P' lies on the circle.

Conversely, show that if the points represented by the complex numbers ab and $\frac{a}{b^*}$, for some non-zero complex number b with $bb^* \neq 1$, both lie on a circle centre C in the Argand diagram which passes through A , then OA is a tangent to the circle.



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