

STEP III, 2005, Q7 MS

7 Substituting $u = x^m$ gives

$$\int \frac{m dx}{x f(x^m)} = \int \frac{m x^{m-1} dx}{x^m f(x^m)} = \int \frac{du}{u f(u)} = F(u) + c = F(x^m) + c$$

(i)
$$\int \frac{dx}{x^n - x} = \int \frac{dx}{x(x^{n-1} - 1)},$$

so letting $u = x^{n-1}$ and $f(u) = u - 1$,

$$\int \frac{(n-1)dx}{x^n - x} = \int \frac{du}{u(u-1)} = \int \frac{1}{u-1} - \frac{1}{u} du = \ln \left| \frac{u-1}{u} \right|$$

so
$$\int \frac{dx}{x^n - x} = \frac{1}{n-1} \ln \left| \frac{x^{n-1} - 1}{x^{n-1}} \right| + c.$$

(ii)
$$\int \frac{dx}{\sqrt{x^n + x^2}} = \int \frac{dx}{x\sqrt{x^{n-2} + 1}} \quad (\text{for } x > 0)$$

so letting $u = x^{n-2}$ and $f(u) = \sqrt{u+1}$ (and assuming $n \neq 2$)

$$\int \frac{(n-2)dx}{\sqrt{x^n + x^2}} = \int \frac{du}{u\sqrt{u+1}}.$$

Substituting $u = v^2 - 1$ with $v > 0$,

$$\int \frac{du}{u\sqrt{u+1}} = \int \frac{2vdv}{(v^2-1)v} = \int \frac{1}{v-1} - \frac{1}{v+1} dv = \ln \left| \frac{v-1}{v+1} \right|$$

so
$$\int \frac{dx}{\sqrt{x^n + x^2}} = \frac{1}{n-2} \ln \left| \frac{\sqrt{x^{n-2} + 1} - 1}{\sqrt{x^{n-2} + 1} + 1} \right| + c.$$



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