

STEP III, 2005, Q3

- 3 Let $f(x) = x^2 + px + q$ and $g(x) = x^2 + rx + s$. Find an expression for $f(g(x))$ and hence find a necessary and sufficient condition on a , b and c for it to be possible to write the quartic expression $x^4 + ax^3 + bx^2 + cx + d$ in the form $f(g(x))$, for some choice of values of p , q , r and s .

Show further that this condition holds if and only if it is possible to write the quartic expression $x^4 + ax^3 + bx^2 + cx + d$ in the form $(x^2 + vx + w)^2 - k$, for some choice of values of v , w and k .

Find the roots of the quartic equation $x^4 - 4x^3 + 10x^2 - 12x + 4 = 0$.



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