

STEP III, 2005, Q2

- 2 Find the general solution of the differential equation $\frac{dy}{dx} = -\frac{xy}{x^2 + a^2}$, where $a \neq 0$, and show that it can be written in the form $y^2(x^2 + a^2) = c^2$, where c is an arbitrary constant. Sketch this curve.

Find an expression for $\frac{d}{dx}(x^2 + y^2)$ and show that

$$\frac{d^2}{dx^2}(x^2 + y^2) = 2 \left(1 - \frac{c^2}{(x^2 + a^2)^2} \right) + \frac{8c^2x^2}{(x^2 + a^2)^3}.$$

- (i) Show that, if $0 < c < a^2$, the points on the curve whose distance from the origin is least are $\left(0, \pm \frac{c}{a}\right)$.
- (ii) If $c > a^2$, determine the points on the curve whose distance from the origin is least.



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