

STEP III, 2005, Q12

- 12 Five independent timers time a runner as she runs four laps of a track. Four of the timers measure the individual lap times, the results of the measurements being the random variables T_1 to T_4 , each of which has variance σ^2 and expectation equal to the true time for the lap. The fifth timer measures the total time for the race, the result of the measurement being the random variable T which has variance σ^2 and expectation equal to the true race time (which is equal to the sum of the four true lap times).

Find a random variable X of the form $aT + b(T_1 + T_2 + T_3 + T_4)$, where a and b are constants independent of the true lap times, with the two properties:

- (1) whatever the true lap times, the expectation of X is equal to the true race time;
- (2) the variance of X is as small as possible.

Find also a random variable Y of the form $cT + d(T_1 + T_2 + T_3 + T_4)$, where c and d are constants independent of the true lap times, with the property that, whatever the true lap times, the expectation of Y^2 is equal to σ^2 .

In one particular race, T takes the value 220 seconds and $(T_1 + T_2 + T_3 + T_4)$ takes the value 220.5 seconds. Use the random variables X and Y to estimate an interval in which the true race time lies.



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