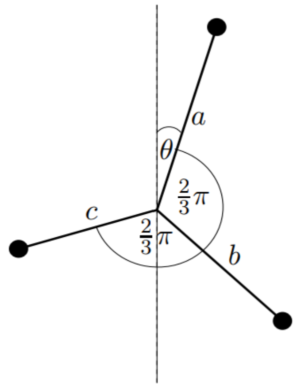
STEP III, 2005, Q11

11 A horizontal spindle rotates freely in a fixed bearing. Three light rods are each attached by one end to the spindle so that they rotate in a vertical plane. A particle of mass m is fixed to the other end of each of the three rods. The rods have lengths a, b and c, with $a > b > c$ and the angle between any pair of rods is $\frac{2}{3}\pi$. The angle between the rod of length a and the vertical is θ, as shown in the diagram.



Find an expression for the energy of the system and show that, if the system is in equilibrium, then

$$\tan\theta = -\frac{(b-c)\sqrt{3}}{2a-b-c}\,.$$

Deduce that there are exactly two equilibrium positions and determine which of the two equilibrium positions is stable.

Show that, for the system to make complete revolutions, it must pass through its position of stable equilibrium with an angular velocity of at least

$$\sqrt{\frac{4gR}{a^2+b^2+c^2}}\,,$$

where $2R^2 = (a-b)^2 + (b-c)^2 + (c-a)^2$.