

STEP III, 2004, Q8 MS

8 If $u = y^2$ then $\frac{du}{dx} = 2y \frac{dy}{dx} = 2f(x)y^2 + 2g(x) = 2f(x)u + 2g(x)$,

which is a linear differential equation for $u(x)$.

In this case, $f(x) = \frac{1}{x}$, $g(x) = -1$ so the differential equation is $\frac{du}{dx} = \frac{2u}{x} - 2$.

The integrating factor is $e^{\int \frac{2}{x} dx} = e^{2 \ln x} = \frac{1}{x^2}$, giving $\frac{1}{x^2} \frac{du}{dx} - \frac{2u}{x^3} = \frac{d}{dx} \left(\frac{u}{x^2} \right) = \frac{-2}{x^2}$

so that $\frac{u}{x^2} = \int \frac{-2}{x^2} dx = \frac{2}{x} + c$ or $u = y^2 = cx^2 + 2x$.

The solution curves which pass through (1, 1), (2, 2) and (4, 4) are $y^2 + (x - 1)^2 = 1$, $y^2 = 2x$ and $(x + 2)^2 - 2y^2 = 4$ respectively. In drawing these curves it should be made clear that all of them pass through the origin, and that this is their only point of intersection; that the first is a circle with centre (1, 0), the second a parabola and the third an hyperbola with centre (-2, 0) and asymptotes $y = \pm \frac{x + 2}{\sqrt{2}}$.



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