

STEP III, 2004, Q7

7 For $n = 1, 2, 3, \dots$, let

$$I_n = \int_0^1 \frac{t^{n-1}}{(t+1)^n} dt.$$

By considering the greatest value taken by $\frac{t}{t+1}$ for $0 \leq t \leq 1$ show that $I_{n+1} < \frac{1}{2}I_n$.

Show also that $I_{n+1} = -\frac{1}{n2^n} + I_n$.

Deduce that $I_n < \frac{1}{n2^{n-1}}$.

Prove that

$$\ln 2 = \sum_{r=1}^n \frac{1}{r2^r} + I_{n+1}$$

and hence show that $\frac{2}{3} < \ln 2 < \frac{17}{24}$.



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