

### STEP III, 2004, Q6 MS

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$$F_n - F_{n-1} = w_n^2 + w_{n-1}^2 - 4w_n w_{n-1} - w_{n-1}^2 - w_{n-2}^2 + 4w_{n-1} w_{n-2} = w_n^2 - w_{n-2}^2 - 4w_{n-1}(w_n - w_{n-2})$$
  

$$= (w_n - w_{n-2})(w_n + w_{n-2} - 4w_{n-1}) \quad (+)$$

(i) Let  $w_n$  be  $u_n$ ; then  $u_n + u_{n-2} - 4u_{n-1} = 0$ , so  $F_n - F_{n-1} = 0$  for  $n \geq 2$ , by (+),  
 but  $F_1 = u_1^2 + u_0^2 - 4u_1 u_0 = -3$  so  $F_n = -3$  for  $n \geq 1$

(ii) In this part, let  $w_n$  be  $v_n$ .

(a)  $v_1^2 + 1 = 4v_1 - 3 \Rightarrow (v_1 - 2)^2 = 0 \Rightarrow v_1 = 2$

$F_n = v_n^2 + v_{n-1}^2 - 4v_n v_{n-1} = -3$  for  $n \geq 1$

$\Rightarrow v_n - v_{n-2} = 0$  or  $v_n + v_{n-2} - 4v_{n-1} = 0$ , for  $n \geq 2$ , by (+).

(b) Since  $1, 2, 1, 2, \dots$  satisfies  $v_n - v_{n-2} = 0$  for  $n \geq 2$ ,  $F_n$  is constant, by (+) and since  $v_0 = 1, v_1 = 2$  that constant is  $-3$ , so the sequence satisfies (\*).

(c) The sequence  $1, 2, 7, 2, \dots$ , with period 4, satisfies  $v_n - v_{n-2} = 0$  for odd  $n \geq 2$  and  $v_n + v_{n-2} - 4v_{n-1} = 0$  for even  $n \geq 2$ , so  $F_n$  is constant, by (+), and since  $v_0 = 1, v_1 = 2$  that constant is  $-3$ , so the sequence satisfies (\*).



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