

## STEP III, 2004, Q6

- 6 Given a sequence  $w_0, w_1, w_2, \dots$ , the sequence  $F_1, F_2, \dots$  is defined by

$$F_n = w_n^2 + w_{n-1}^2 - 4w_n w_{n-1}.$$

Show that  $F_n - F_{n-1} = (w_n - w_{n-2})(w_n + w_{n-2} - 4w_{n-1})$  for  $n \geq 2$ .

- (i) The sequence  $u_0, u_1, u_2, \dots$  has  $u_0 = 1$ , and  $u_1 = 2$  and satisfies

$$u_n = 4u_{n-1} - u_{n-2} \quad (n \geq 2).$$

Prove that  $u_n^2 + u_{n-1}^2 = 4u_n u_{n-1} - 3$  for  $n \geq 1$ .

- (ii) A sequence  $v_0, v_1, v_2, \dots$  has  $v_0 = 1$  and satisfies

$$v_n^2 + v_{n-1}^2 = 4v_n v_{n-1} - 3 \quad (n \geq 1). \quad (*)$$

(a) Find  $v_1$  and prove that, for each  $n \geq 2$ , either  $v_n = 4v_{n-1} - v_{n-2}$  or  $v_n = v_{n-2}$ .

(b) Show that the sequence, with period 2, defined by

$$v_n = \begin{cases} 1 & \text{for } n \text{ even} \\ 2 & \text{for } n \text{ odd} \end{cases}$$

satisfies (\*).

(c) Find a sequence  $v_n$  with period 4 which has  $v_0 = 1$ , and satisfies (\*).



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