

STEP III, 2004, Q5 MS

5 If $\cos(x - \alpha) = \cos \beta$ then $x - \alpha = 2n\pi \pm \beta$ so $x = \alpha \pm \beta + 2n\pi$ so $\tan x = \tan(\alpha \pm \beta)$ however, for example, $x = \pi$, $\alpha = \beta = 0$ has $\tan x = \tan \pi = \tan 0 = \tan(\alpha + \beta)$ but $\cos(x - \alpha) = \cos \pi = -1 \neq 1 = \cos \beta$.

a Writing $\cos x - 7 \sin x = R \cos(x - \alpha)$ requires $R = \sqrt{50} = 5\sqrt{2}$ and $\tan \alpha = -7$, so $\cos(x - \alpha) = \cos \beta$, where $\cos \beta = \frac{1}{\sqrt{2}}$, so we can take $\tan \beta = 1$.

$$\text{Hence } \tan x = \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{-7 \pm 1}{1 \pm 7} = -\frac{3}{4} \text{ or } \frac{4}{3}.$$

The first of these gives $x = \frac{1}{2}\pi + \omega$ or $x = \frac{3}{2}\pi + \omega$ (since $\arctan \frac{3}{4} = \frac{\pi}{2} - \arctan \frac{4}{3}$) and the second $x = \omega$ or $x = \pi + \omega$. However, the first solution in each case does not satisfy the original equation (both have $\sin x > 0$, so $\cos x - 7 \sin x < 1$), so $x = \frac{3}{2}\pi + \omega$ or $\pi + \omega$.

b proceeding as in (i), $\cos(x - \alpha) = \cos \beta$, where $\tan \alpha = \frac{11}{2}$ and $R = 5\sqrt{5}$, so $\cos \beta = \frac{2}{\sqrt{5}}$ and so $\tan \beta = \frac{1}{2}$. Hence $\tan x = \frac{22 \pm 2}{4 \mp 11} = -\frac{24}{7}$ or $\frac{4}{3}$.

Notice that $\tan 2\omega = \frac{2 \tan \omega}{1 - \tan^2 \omega} = -\frac{24}{7}$ so the solutions are $x = \omega$ and $x = 2\omega$, again eliminating the other two possibilities, $\omega + \pi$ and $2\omega + \pi$, by checking in the original equation.



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