

### STEP III, 2004, Q3

- 3 Given that  $f''(x) > 0$  when  $a \leq x \leq b$ , explain with the aid of a sketch why

$$(b - a) f\left(\frac{a + b}{2}\right) < \int_a^b f(x) \, dx < (b - a) \frac{f(a) + f(b)}{2}.$$

By choosing suitable  $a$ ,  $b$  and  $f(x)$ , show that

$$\frac{4}{(2n - 1)^2} < \frac{1}{n - 1} - \frac{1}{n} < \frac{1}{2} \left( \frac{1}{n^2} + \frac{1}{(n - 1)^2} \right),$$

where  $n$  is an integer greater than 1.

Deduce that

$$4 \left( \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots \right) < 1 < \frac{1}{2} + \left( \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \right).$$

Show that

$$\frac{1}{2} \left( \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots \right) < \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$$

and hence show that

$$\frac{3}{2} < \sum_{n=1}^{\infty} \frac{1}{n^2} < \frac{7}{4}.$$



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