

## STEP III, 2004, Q14 MS

- 14 The test is appropriate because, if  $H_0$  were true,  $\bar{x}$  would have a higher probability of being in the region stated than if  $H_1$  were true.

Under  $H_0$ ,  $\bar{X}$  has a Normal distribution with mean  $\mu$  and standard deviation  $\frac{\sigma_0}{\sqrt{n}}$  so

$$\alpha = P(|\bar{X} - \mu| > c) = 2 \left( 1 - \Phi \left( \frac{c}{\frac{\sigma_0}{\sqrt{n}}} \right) \right)$$

so  $\Phi \left( \frac{c}{\frac{\sigma_0}{\sqrt{n}}} \right) = 1 - \frac{\alpha}{2}$ , so  $\frac{c}{\frac{\sigma_0}{\sqrt{n}}} = z_\alpha$  or  $c = \frac{\sigma_0 z_\alpha}{\sqrt{n}}$ .

Under  $H_1$ ,  $\bar{X}$  has a Normal distribution with mean  $\mu$  and standard deviation  $\frac{\sigma_1}{\sqrt{n}}$  so

$$\beta = P(|\bar{X} - \mu| < c) = 1 - 2 \left( 1 - \Phi \left( \frac{c}{\frac{\sigma_1}{\sqrt{n}}} \right) \right) = 2\Phi \left( \frac{c}{\frac{\sigma_1}{\sqrt{n}}} \right) - 1 = 2\Phi \left( \frac{\sigma_0 z_\alpha}{\sigma_1} \right) - 1,$$

so  $\beta$  is independent of  $n$ .

$$\beta < 0.05 \Rightarrow \Phi \left( \frac{\sigma_0 z_\alpha}{\sigma_1} \right) < \frac{1 + 0.05}{2} = 0.525 \Rightarrow \frac{\sigma_0 z_\alpha}{\sigma_1} < 0.063.$$

$$\alpha < 0.05 \Rightarrow z_\alpha > 1.960.$$

For these both to hold, we must have  $0.063 > \frac{\sigma_0 z_\alpha}{\sigma_1} > \frac{1.960\sigma_0}{\sigma_1}$  or  $\frac{\sigma_1}{\sigma_0} > \frac{1.960}{.063} = \frac{280}{9} \approx 30$ .



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