STEP III, 2004, Q14

14 In this question, $\Phi(z)$ is the cumulative distribution function of a standard normal random variable.

A random variable is known to have a Normal distribution with mean μ and standard deviation either σ_0 or σ_1, where $\sigma_0 < \sigma_1$. The mean, $\overline{X}$, of a random sample of n values of X is to be used to test the hypothesis $\mathrm{H}_0 : \sigma = \sigma_0$ against the alternative $\mathrm{H}_1 : \sigma = \sigma_1$.

Explain carefully why it is appropriate to use a two sided test of the form: accept H_0 if $\mu - c < \overline{X} < \mu + c$, otherwise accept H_1.

Given that the probability of accepting H_1 when H_0 is true is α, determine c in terms of n, σ_0 and z_α, where z_α is defined by $\Phi(z_\alpha) = 1 - \dfrac{\alpha}{2}$.

The probability of accepting H_0 when H_1 is true is denoted by β. Show that β is independent of n.

Given that $\Phi(1.960) \approx 0.975$ and that $\Phi(0.063) \approx 0.525$, determine, approximately, the minimum value of $\dfrac{\sigma_1}{\sigma_0}$ if α and β are both to be less than 0.05.