

STEP III, 2004, Q12

- 12 A team of m players, numbered from 1 to m , puts on a set of a m shirts, similarly numbered from 1 to m . The players change in a hurry, so that the shirts are assigned to them randomly, one to each player.

Let C_i be the random variable that takes the value 1 if player i is wearing shirt i , and 0 otherwise. Show that $E[C_1] = \frac{1}{m}$ and find $\text{Var}[C_1]$ and $\text{Cov}[C_1, C_2]$.

Let $N = C_1 + C_2 + \cdots + C_m$ be the random variable whose value is the number of players who are wearing the correct shirt. Show that $E[N] = \text{Var}[N] = 1$.

Explain why a Normal approximation to N is not likely to be appropriate for any m , but that a Poisson approximation might be reasonable.

In the case $m = 4$, find, by listing equally likely possibilities or otherwise, the probability that no player is wearing the correct shirt and verify that an appropriate Poisson approximation to N gives this probability with a relative error of about 2%. [Use $e \approx 2\frac{72}{100}$.]



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