

STEP III, 2004, Q1

1 Show that

$$\int_0^a \frac{\sinh x}{2 \cosh^2 x - 1} dx = \frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{2} \cosh a - 1}{\sqrt{2} \cosh a + 1} \right) + \frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)$$

and find

$$\int_0^a \frac{\cosh x}{1 + 2 \sinh^2 x} dx.$$

Hence show that

$$\int_0^\infty \frac{\cosh x - \sinh x}{1 + 2 \sinh^2 x} dx = \frac{\pi}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right).$$

By substituting $u = e^x$ in this result, or otherwise, find

$$\int_1^\infty \frac{1}{1 + u^4} du.$$



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