STEP III, 2003 Q7

7 In the x–y plane, the point A has coordinates $(a\,,0)$ and the point B has coordinates $(0\,,b)$, where a and b are positive. The point P, which is distinct from A and B, has coordinates (s,t). X and Y are the feet of the perpendiculars from P to the x–axis and y–axis respectively, and N is the foot of the perpendicular from P to the line AB. Show that the coordinates $(x\,,y)$ of N are given by

$$x = \frac{ab^2 - a(bt - as)}{a^2 + b^2}\,, \quad y = \frac{a^2 b + b(bt - as)}{a^2 + b^2}\,.$$

Show that, if $\left(\frac{t-b}{s}\right)\left(\frac{t}{s-a}\right) = -1$, then N lies on the line XY.

Give a geometrical interpretation of this result.

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