

## STEP III, 2003 Q2

2 Show that  ${}^{2r}C_r = \frac{1 \times 3 \times \cdots \times (2r-1)}{r!} \times 2^r$ , for  $r \geq 1$ .

(i) Give the first four terms of the binomial series for  $(1-p)^{-\frac{1}{2}}$ .

By choosing a suitable value for  $p$  in this series, or otherwise, show that

$$\sum_{r=0}^{\infty} \frac{{}^{2r}C_r}{8^r} = \sqrt{2}.$$

(ii) Show that

$$\sum_{r=0}^{\infty} \frac{(2r+1) {}^{2r}C_r}{5^r} = (\sqrt{5})^3.$$

[**Note:**  ${}^nC_r$  is an alternative notation for  $\binom{n}{r}$  for  $r \geq 1$ , and  ${}^0C_0 = 1$ .]



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