STEP III, 2003 Q13

13 In a rabbit warren, underground chambers A, B, C and D are at the vertices of a square, and burrows join A to B, B to C, C to D and D to A. Each of the chambers also has a tunnel to the surface. A rabbit finding itself in any chamber runs along one of the two burrows to a neighbouring chamber, or leaves the burrow through the tunnel to the surface. Each of these three possibilities is equally likely.

Let p_A, p_B, p_C and p_D be the probabilities of a rabbit leaving the burrow through the tunnel from chamber A, given that it is currently in chamber A, B, C or D, respectively.

(i) Explain why $p_A = \frac{1}{3} + \frac{1}{3}p_B + \frac{1}{3}p_D$.

(ii) Determine p_A.

(iii) Find the probability that a rabbit which starts in chamber A does not visit chamber C, given that it eventually leaves the burrow through the tunnel in chamber A.