

STEP III, 2002 Q9

- 9 A tall container made of light material of negligible thickness has the form of a prism, with a square base of area a^2 . It contains a volume ka^3 of fluid of uniform density. The container is held so that it stands on a rough plane, which is inclined at angle θ to the horizontal, with two of the edges of the base of the container horizontal. In the case $k > \frac{1}{2} \tan \theta$, show that the centre of mass of the fluid is at a distance x from the lower side of the container and at a distance y from the base of the container, where

$$\frac{x}{a} = \frac{1}{2} - \frac{\tan \theta}{12k}, \quad \frac{y}{a} = \frac{k}{2} + \frac{\tan^2 \theta}{24k}.$$

Determine the corresponding coordinates in the case $k < \frac{1}{2} \tan \theta$.

The container is now released. Given that $k < \frac{1}{2}$, show that the container will topple if $\theta > 45^\circ$.



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