

## STEP III, 2002 Q8

- 8 Four complex numbers  $u_1, u_2, u_3$  and  $u_4$  have unit modulus, and arguments  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$ , respectively, with  $-\pi < \theta_1 < \theta_2 < \theta_3 < \theta_4 < \pi$ .

Show that

$$\arg(u_1 - u_2) = \frac{1}{2}(\theta_1 + \theta_2 - \pi) + 2n\pi$$

where  $n = 0$  or  $1$ . Deduce that

$$\arg((u_1 - u_2)(u_4 - u_3)) = \arg((u_1 - u_4)(u_3 - u_2)) + 2n\pi$$

for some integer  $n$ .

Prove that

$$|(u_1 - u_2)(u_4 - u_3)| + |(u_1 - u_4)(u_3 - u_2)| = |(u_1 - u_3)(u_4 - u_2)|.$$



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