

## STEP III, 2002 Q7

- 7 Given that  $\alpha$  and  $\beta$  are acute angles, show that  $\alpha + \beta = \pi/2$  if and only if  $\cos^2 \alpha + \cos^2 \beta = 1$ .

In the  $x$ - $y$  plane, the point  $A$  has coordinates  $(0, s)$  and the point  $C$  has coordinates  $(s, 0)$ , where  $s > 0$ . The point  $B$  lies in the first quadrant ( $x > 0, y > 0$ ). The lengths of  $AB$ ,  $OB$  and  $CB$  are respectively  $a$ ,  $b$  and  $c$ .

Show that

$$(s^2 + b^2 - a^2)^2 + (s^2 + b^2 - c^2)^2 = 4s^2b^2$$

and hence that

$$(2s^2 - a^2 - c^2)^2 + (2b^2 - a^2 - c^2)^2 = 4a^2c^2 .$$

Deduce that

$$(a - c)^2 \leq 2b^2 \leq (a + c)^2 .$$



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