

STEP III, 2002 Q2

- 2 Prove that $\arctan a + \arctan b = \arctan \left(\frac{a+b}{1-ab} \right)$ when $0 < a < 1$ and $0 < b < 1$.

Prove by induction that, for $n \geq 1$,

$$\sum_{r=1}^n \arctan \left(\frac{1}{r^2 + r + 1} \right) = \arctan \left(\frac{n}{n+2} \right)$$

and hence find

$$\sum_{r=1}^{\infty} \arctan \left(\frac{1}{r^2 + r + 1} \right).$$

Hence prove that

$$\sum_{r=1}^{\infty} \arctan \left(\frac{1}{r^2 - r + 1} \right) = \frac{\pi}{2}.$$



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