

STEP III, 2001 Q7

- 7 Sketch the graph of the function $\ln x - \frac{1}{2}x^2$.

Show that the differential equation

$$\frac{dy}{dx} = \frac{2xy}{x^2 - 1}$$

describes a family of parabolas each of which passes through the points $(1, 0)$ and $(-1, 0)$ and has its vertex on the y -axis.

Hence find the equation of the curve that passes through the point $(1, 1)$ and intersects each of the above parabolas orthogonally. Sketch this curve.

[Two curves intersect *orthogonally* if their tangents at the point of intersection are perpendicular.]



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