

STEP III, 2001 Q6

6 The plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

meets the co-ordinate axes at the points A , B and C . The point M has coordinates $(\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c)$ and O is the origin.

Show that OM meets the plane at the centroid $(\frac{1}{3}a, \frac{1}{3}b, \frac{1}{3}c)$ of triangle ABC . Show also that the perpendiculars to the plane from O and from M meet the plane at the orthocentre and at the circumcentre of triangle ABC respectively.

Hence prove that the centroid of a triangle lies on the line segment joining its orthocentre and circumcentre, and that it divides this line segment in the ratio 2 : 1.

[The *orthocentre* of a triangle is the point at which the three altitudes intersect; the *circumcentre* of a triangle is the point equidistant from the three vertices.]



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