

STEP III, 2001 Q4

- 4 In this question, the function \sin^{-1} is defined to have domain $-1 \leq x \leq 1$ and range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and the function \tan^{-1} is defined to have the real numbers as its domain and range $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

- (i) Let

$$g(x) = \frac{2x}{1+x^2}, \quad -\infty < x < \infty.$$

Sketch the graph of $g(x)$ and state the range of g .

- (ii) Let

$$f(x) = \sin^{-1} \left(\frac{2x}{1+x^2} \right), \quad -\infty < x < \infty.$$

Show that $f(x) = 2 \tan^{-1} x$ for $-1 \leq x \leq 1$ and $f(x) = \pi - 2 \tan^{-1} x$ for $x \geq 1$.

Sketch the graph of $f(x)$.



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