STEP III, 2001 Q14

14 A random variable X is distributed uniformly on $[0, a]$. Show that the variance of X is $\frac{1}{12}a^2$.

A sample, X_1 and X_2, of two independent values of the random variable is drawn, and the variance V of the sample is determined. Show that $V = \frac{1}{4}(X_1 - X_2)^2$, and hence prove that $2V$ is an unbiased estimator of the variance of X.

Find an exact expression for the probability that the value of V is less than $\frac{1}{12}a^2$ and estimate the value of this probability correct to one significant figure.