

## **STEP III, 2001 Q13**

- 13** In a game for two players, a fair coin is tossed repeatedly. Each player is assigned a sequence of heads and tails and the player whose sequence appears first wins. Four players,  $A$ ,  $B$ ,  $C$  and  $D$  take turns to play the game. Each time they play,  $A$  is assigned the sequence TTH (i.e. Tail then Tail then Head),  $B$  is assigned THH,  $C$  is assigned HHT and  $D$  is assigned HTT.
- (i)  $A$  and  $B$  play the game. Let  $p_{HH}$ ,  $p_{HT}$ ,  $p_{TH}$  and  $p_{TT}$  be the probabilities of  $A$  winning the game given that the first two tosses of the coin show HH, HT, TH and TT, respectively. Explain why  $p_{TT} = 1$ , and why  $p_{HT} = \frac{1}{2}p_{TH} + \frac{1}{2}p_{TT}$ . Show that  $p_{HH} = p_{HT} = \frac{2}{3}$  and that  $p_{TH} = \frac{1}{3}$ . Deduce that the probability that  $A$  wins the game is  $\frac{2}{3}$ .
- (ii)  $B$  and  $C$  play the game. Find the probability that  $B$  wins.
- (iii) Show that if  $C$  plays  $D$ , then  $C$  is more likely to win than  $D$ , but that if  $D$  plays  $A$ , then  $D$  is more likely to win than  $A$ .



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