

STEP III, 2001 Q10

- 10 Three ships A , B and C move with velocities \mathbf{v}_1 , \mathbf{v}_2 and \mathbf{u} respectively. The velocities of A and B relative to C are equal in magnitude and perpendicular. Write down conditions that \mathbf{u} , \mathbf{v}_1 and \mathbf{v}_2 must satisfy and show that

$$\left| \mathbf{u} - \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) \right|^2 = \left| \frac{1}{2}(\mathbf{v}_1 - \mathbf{v}_2) \right|^2$$

and

$$\left(\mathbf{u} - \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) \right) \cdot (\mathbf{v}_1 - \mathbf{v}_2) = 0.$$

Explain why these equations determine, for given \mathbf{v}_1 and \mathbf{v}_2 , two possible velocities for C , provided $\mathbf{v}_1 \neq \mathbf{v}_2$.

If \mathbf{v}_1 and \mathbf{v}_2 are equal in magnitude and perpendicular, show that if $\mathbf{u} \neq \mathbf{0}$ then $\mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$.



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