

## STEP III, 2001 Q1

- 1 Given that  $y = \ln(x + \sqrt{x^2 + 1})$ , show that  $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$ .

Prove by induction that, for  $n \geq 0$ ,

$$(x^2 + 1)y^{(n+2)} + (2n + 1)xy^{(n+1)} + n^2y^{(n)} = 0,$$

where  $y^{(n)} = \frac{d^n y}{dx^n}$  and  $y^{(0)} = y$ .

Using this result in the case  $x = 0$ , or otherwise, show that the Maclaurin series for  $y$  begins

$$x - \frac{x^3}{6} + \frac{3x^5}{40}$$

and find the next non-zero term.



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