

## STEP III, 2000 Q7

7 Given that

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{r!} + \cdots,$$

use the binomial theorem to show that

$$\left(1 + \frac{1}{n}\right)^n < e$$

for any positive integer  $n$ .

The product  $P(n)$  is defined, for any positive integer  $n$ , by

$$P(n) = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{9}{8} \cdot \cdots \cdot \frac{2^n + 1}{2^n}.$$

Use the arithmetic-geometric mean inequality,

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq (a_1 \cdot a_2 \cdot \cdots \cdot a_n)^{\frac{1}{n}},$$

to show that  $P(n) < e$  for all  $n$ .

Explain briefly why  $P(n)$  tends to a limit as  $n \rightarrow \infty$ . Show that this limit,  $L$ , satisfies  $2 < L \leq e$ .



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