

## STEP III, 2000 Q5

- 5 Given two non-zero vectors  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$  we define  $\Delta(\mathbf{a}, \mathbf{b})$  by  $\Delta(\mathbf{a}, \mathbf{b}) = a_1b_2 - a_2b_1$ .

Let  $A$ ,  $B$  and  $C$  be points with position vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ , respectively, no two of which are parallel. Let  $P$ ,  $Q$  and  $R$  be points with position vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$ , respectively, none of which are parallel.

- (i) Show that there exists a  $2 \times 2$  matrix  $\mathbf{M}$  such that  $P$  and  $Q$  are the images of  $A$  and  $B$  under the transformation represented by  $\mathbf{M}$ .
- (ii) Show that  $\Delta(\mathbf{a}, \mathbf{b})\mathbf{c} + \Delta(\mathbf{c}, \mathbf{a})\mathbf{b} + \Delta(\mathbf{b}, \mathbf{c})\mathbf{a} = \mathbf{0}$ .

Hence, or otherwise, prove that a necessary and sufficient condition for the points  $P$ ,  $Q$ , and  $R$  to be the images of points  $A$ ,  $B$  and  $C$  under the transformation represented by some  $2 \times 2$  matrix  $\mathbf{M}$  is that

$$\Delta(\mathbf{a}, \mathbf{b}) : \Delta(\mathbf{b}, \mathbf{c}) : \Delta(\mathbf{c}, \mathbf{a}) = \Delta(\mathbf{p}, \mathbf{q}) : \Delta(\mathbf{q}, \mathbf{r}) : \Delta(\mathbf{r}, \mathbf{p}).$$



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