

STEP III, 2000 Q4

- 4 The function $f(x)$ is defined by

$$f(x) = \frac{x(x-2)(x-a)}{x^2-1}.$$

Prove algebraically that the line $y = x + c$ intersects the curve $y = f(x)$ if $|a| \geq 1$, but there are values of c for which there are no points of intersection if $|a| < 1$.

Find the equation of the oblique asymptote of the curve $y = f(x)$. Sketch the graph in the two cases (i) $a < -1$; and (ii) $-1 < a < -\frac{1}{2}$. (You need not calculate the turning points.)



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