

## **STEP III, 2000 Q14**

- 14 The random variable  $X$  takes only the values  $x_1$  and  $x_2$  (where  $x_1 \neq x_2$ ), and the random variable  $Y$  takes only the values  $y_1$  and  $y_2$  (where  $y_1 \neq y_2$ ). Their joint distribution is given by

$$P(X = x_1, Y = y_1) = a ; \quad P(X = x_1, Y = y_2) = q - a ; \quad P(X = x_2, Y = y_1) = p - a .$$

Show that if  $E(XY) = E(X)E(Y)$  then

$$(a - pq)(x_1 - x_2)(y_1 - y_2) = 0.$$

Hence show that two random variables each taking only two distinct values are independent if  $E(XY) = E(X)E(Y)$ .

Give a joint distribution for two random variables  $A$  and  $B$ , each taking the three values  $-1$ ,  $0$  and  $1$  with probability  $\frac{1}{3}$ , which have  $E(AB) = E(A)E(B)$ , but which are not independent.



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