

STEP III, 1999, Q8

- 8 The function $y(x)$ is defined for $x \geq 0$ and satisfies the conditions

$$y = 0 \quad \text{and} \quad \frac{dy}{dx} = 1 \quad \text{at} \quad x = 0.$$

When x is in the range $2(n-1)\pi < x < 2n\pi$, where n is a positive integer, $y(x)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + n^2y = 0.$$

Both y and $\frac{dy}{dx}$ are continuous at $x = 2n\pi$ for $n = 0, 1, 2, \dots$

- (i) Find $y(x)$ for $0 \leq x \leq 2\pi$.
- (ii) Show that $y(x) = \frac{1}{2} \sin 2x$ for $2\pi \leq x \leq 4\pi$, and find $y(x)$ for all $x \geq 0$.
- (iii) Show that

$$\int_0^{\infty} y^2 dx = \pi \sum_{n=1}^{\infty} \frac{1}{n^2}.$$



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